Teaching Universal Gravitation with Vector Games

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Like many high school and college physics teachers, I have found playing vector games to be a useful way of illustrating the concepts of inertia, velocity, and acceleration. Like many, I have also had difficulty in trying to get students to understand Newton’s law of universal gravitation, specifically the inverse-square law and its application to motion. In this paper, I’ll outline a way to address this problem through use of a vector game. The inspiration for this idea came from a January 1998 article in *The Physics Teacher* by Michael Vinson entitled “Space Race: A Game of Physics Adventure.”

The basic idea of Vinson’s Space Race is to start with a grid of standard graph paper and have students imagine that this grid represents a section of two-dimensional space in which the race will occur. The players move in turns that represent a fixed amount of time. The motion of the players’ ships is shown by a displacement vector for that time (the ship’s average velocity during the interval). The rules outlining the game are as follows:

1. Each player has a ship that starts out with a velocity of zero.
2. Players may, during their turn, change their velocity—in each direction (x and y)—by one unit per turn. This is due to the firing of thrusters on their ship.
3. Note that the motion in each direction (x and y) is independent of the other. If a player opts for no change in velocity, they continue moving just as they did on the previous turn, illustrating the law of inertia.
4. If a ship ends its turn by impacting an obstacle such as an asteroid, energy barrier, or other ship, then that player’s ship is destroyed. Excessive acceleration and lack of attention are likely to result in such a crash.
5. The goal is to win the race, usually by reaching a finish line somewhere on the grid. The player reaching the finish line in the fewest moves wins.

An example of a simple race is illustrated in Fig. 1.

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**Fig. 1. A hypothetical start to the Space Race, showing a ship starting from rest (red dot) on the starting line. The brown spots are all the possible moves available to this ship on the first turn.**
The ship starts out at a velocity of (0,0) on the starting line—(0,0) represents the x- and y-components of the ship’s velocity. On his first turn, the player may opt for a variety of moves—for example, he may move straight forward with a velocity of (1,0) (one square to the right); he may choose to move to the side with a velocity of (0,1) (up) or (0,-1) (down); or he can move diagonally with a velocity of (1,1) (up and straight) or (1,-1) (down and straight). Since the race is basically a sprint through space, there are no limits on how much velocity may be achieved (there is no air resistance in vacuum) or how long a ship may accelerate (assume the ships’ fuel tanks are sufficiently large).

Suppose that our student chooses to move with a velocity of (1,0) on his first turn. What options are available to him on turn #2? In Fig. 2, the options are illustrated. The player may choose to simply coast by having no change in velocity; he may speed up in the same direction, changing his velocity to (2,0), which means he would then move two squares to the right; he could decide to slow down to a stop, changing his velocity to (0,0) again; or he could speed up while moving to the side by moving (2,1) (speed up and move up) or (2,-1) (speed up and move down). A few minutes practice with these simple rules is enough for most students to get the hang of the game. If there are any disagreements between players, I usually act as arbiter to make sure they are complying with the laws of physics.

Early in the semester, I use the Space Race as a way to illustrate various aspects of motion—inertia, velocity, and acceleration. It’s very fun, and the students enjoy it very much; I often like to tease them that I’ve fooled them into learning physics. Due to the simplicity of the rules and the ease of making up one’s own grids (all you need is graph paper and a marker), many students have made up their own races and played the game outside of class!

I also use the popular idea of the Space Race as a way to get students to understand the inverse-square law of gravity. So I make some simple adjustments to the game and present it again to my classes later in the semester during the unit on universal gravitation, just after I’ve introduced the students to the concept of the inverse-square law of gravity, including the formula \( F = \frac{-GMm}{r^2} \).

My adjusted version of the Space Race is what I call “The Black Hole Space Race”—the difference is that
in the exact center of the grid is a small black dot, representing a black hole. In this version, all the previous rules for the motion apply, with one addition—after a ship moves, the player must take into account the effect of the black hole’s gravity on her ship. Here’s how it is done:

1. The student uses a metric ruler to measure the straight-line distance (in centimeters) from her ship’s location (after it is moved) to the black hole.

2. This distance is then squared, so a measured distance of \( d = 3.0 \text{ cm} \) would be \( d^2 = 9.0 \).

3. Divide an arbitrary number that represents the inherent “strength” of the black hole’s gravity by the number from step 2—in our example we use 75, so we’d get \( 75/9 = 8.33 \). This is the number of additional squares the player must move her ship directly toward the black hole.

4. The player lightly draws a straight line between the current position of her ship and the black hole, and she moves her ship toward the black hole by the number of squares equal to the result from step 3. The ship starts its next move at the grid point closest to that resulting position.

5. Note that the motion of the ship (as a result of the use of thrusters) is independent of the gravitational effects from the black hole.

Needless to say, this adjustment takes a little getting used to. Let’s work through an example as outlined in Fig. 3. Let us suppose a player’s ship is moving along with a velocity of \((3, -4)\) and he wishes to move \((4, -3)\) on his next turn. The player first moves his ship in accordance to the usual rules. The student then applies the new rules for the black hole’s gravity and measures a distance from the ship’s current location to the black hole of 7.4 cm. The calculation of 75/7.4\(^2\) yields a result of 1.36; if we round this to 1.4, we see that gravity drags the ship by 1.4 squares directly towards the black hole. The player places the ship at the nearest grid point, and this position is the new position from which the ship will begin its next turn.

Many times, students are quite surprised the first time they play this version of the Space Race. Most of them don’t expect the gravity of the black hole to affect the motion of their ship too much; it isn’t until they get too close to the black hole that they truly begin to appreciate the power of the inverse-square law. The usual result is that ships either get dragged into the black hole itself or into another obstacle and are thus destroyed. However, students can learn to incorporate the black hole’s gravity into the race and use the gravitational pull to their advantage. I’ve even seen students perform the equivalent of a gravitational assist or “slingshot,” such as that commonly performed by interplanetary space probes, in their race in order to make a tight turn or pull ahead of their opponents!

One can make additional adjustments to the game. One possibility is to change the mass of the black hole by adjusting the value in step 3 of the gravity calculation. For example, using 100 rather than 75 would indicate a more massive, and much more deadly, black hole than before. So the linear dependence of the gravitational force on mass can also be illustrated using this exercise.

This is one of the most popular activities that I have my students work on in class, and they often complain when I insist that we must move on to other topics. By incorporating the concepts of the inverse-square law of gravity into this simple game, I have had great success at getting the students to understand at a much more conceptual level the power of gravity. This exercise is also very useful for students who are less inclined toward mathematics since it presents the math in a fun and less threatening manner than seen in the typical classroom. By introducing the Black Hole Space Race early in the unit on universal gravity, I can build upon those concepts toward more complicated scenarios. Not to mention, this method has the added advantage that it is really fun!

Reference

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