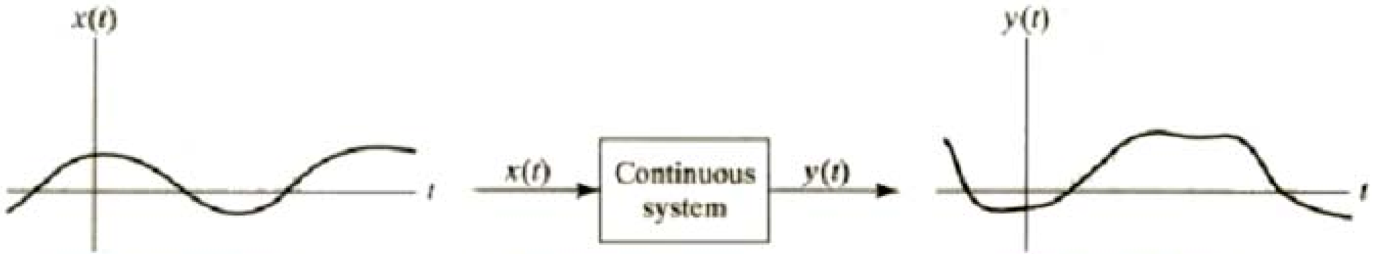
선 형 변 환

1장 Signal and Sequence

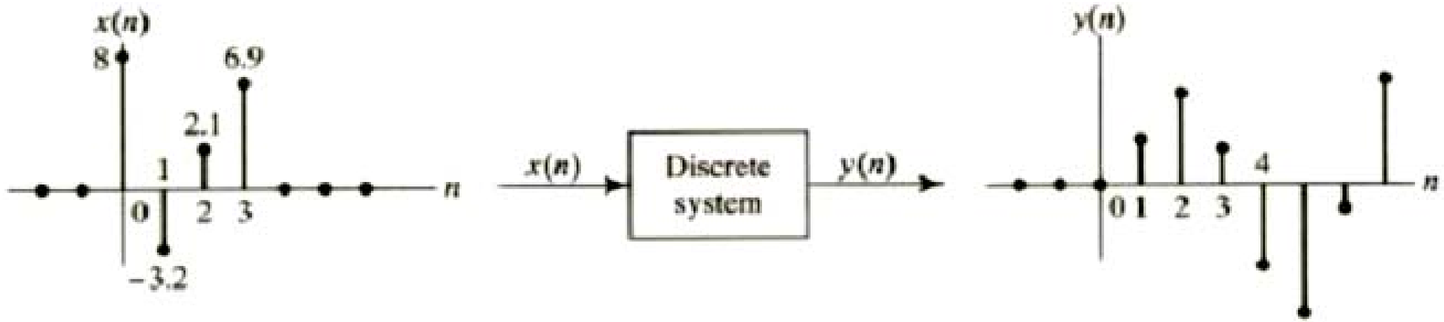
1. Continuous and Discrete System

Continuous system



a. Analog input signal   b. Continuous system   c. Analog output signal

Discrete system



a. Input sequence   b. Discrete system   c. Output sequence

2. Continuous Time Signals

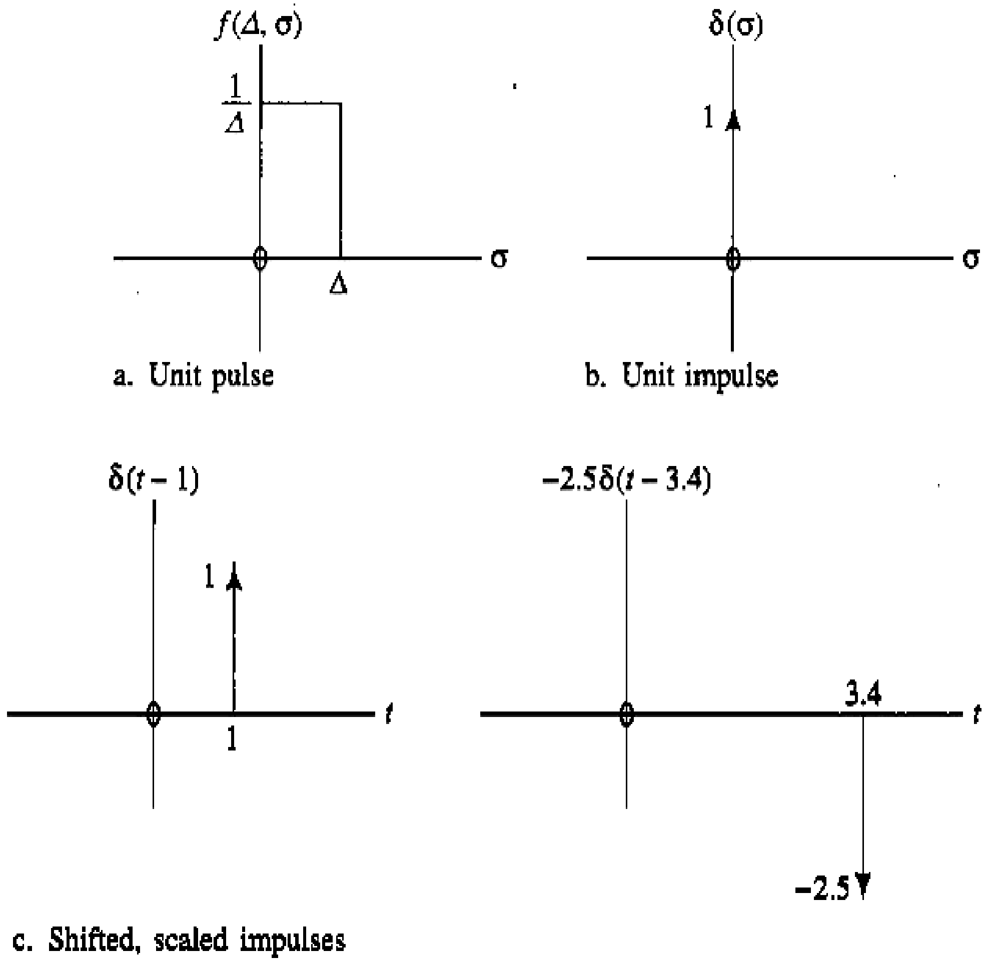
a. Unit Impulse

Definition

$$\delta(\sigma) = \lim_{\Delta \to 0} f(\Delta, \sigma)$$

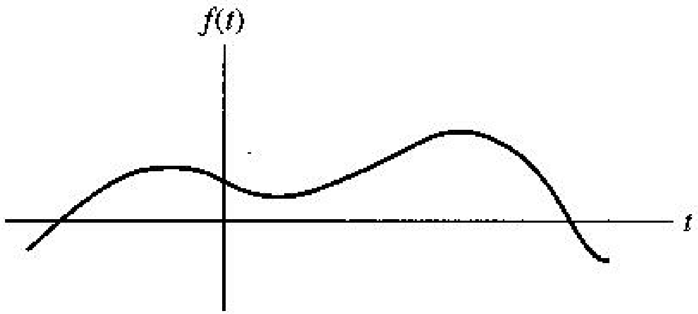
Sifting Property

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_1)dt = f(t_1)$$

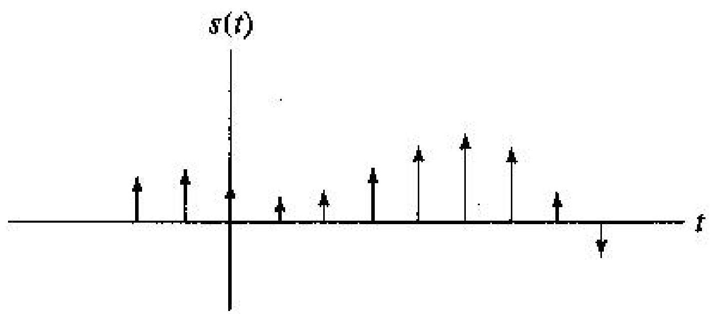


b. Arbitrary Function

There is no analytical way of exactly describing and arbitrary analog waveshape such as the one in Figure(a), however, equally spaced impulses can be used as an approximate representation, Figure(b).



a. Arbitrary function



b. Approximation of sum of impulses

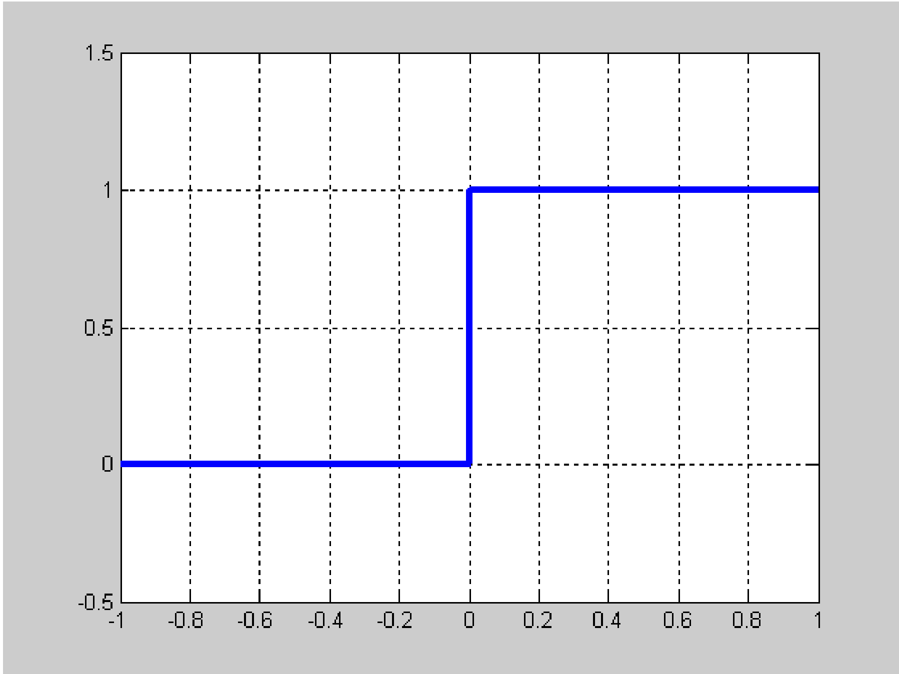
As, the number of impulses becomes infinite

$$f(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau$$

c. Unit Step Function

Definition

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

Generic Step Function

$$Bu(t - t_0) = \begin{cases} B, & t \geq t_0 \\ 0, & t < t_0 \end{cases}$$

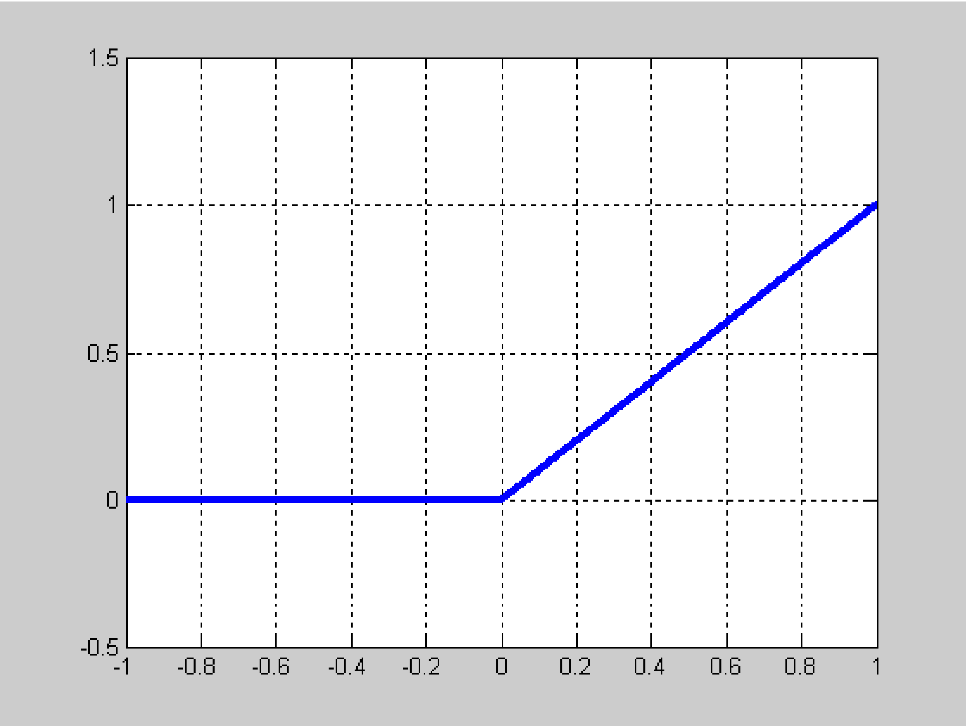
d. Ramp Function

Definition

$$g(t) = b(t - t_0)$$

Unit Ramp Function

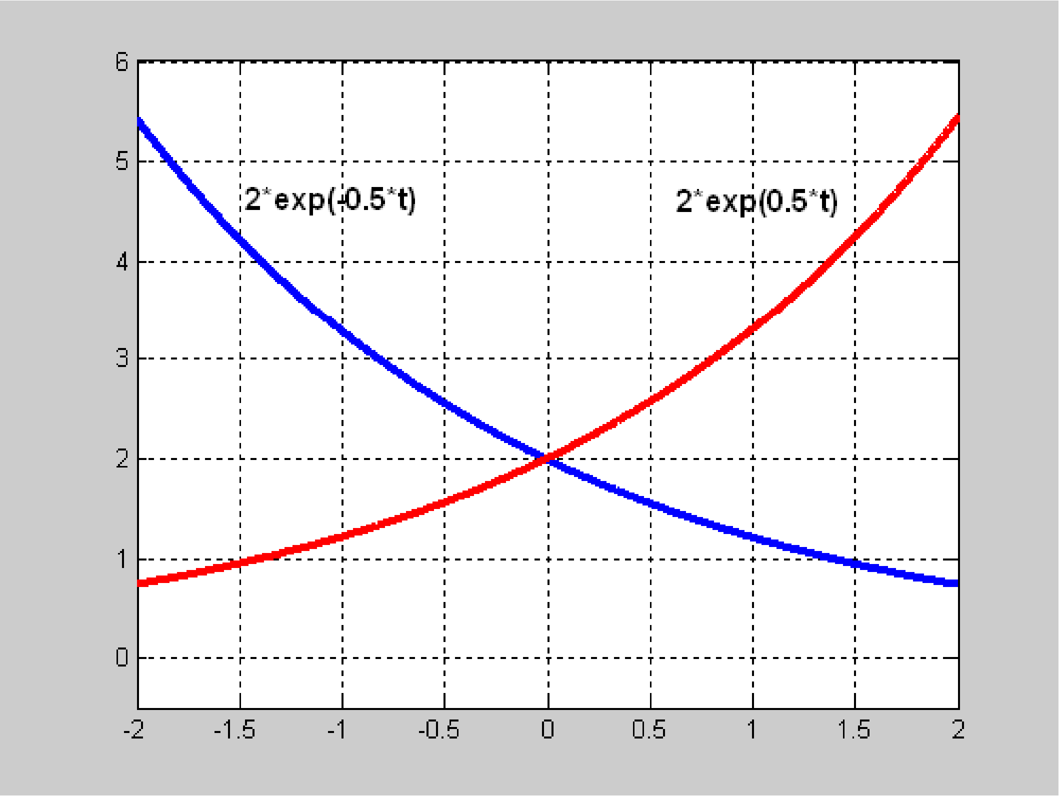
$$r(t) = tu(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$



e. Real Exponential Function

Definition

$$f(t) = Ae^{at}$$



f. Sinusoidal Function

$$f(t) = A\cos(2\pi f t + \alpha) = A\cos(\omega t + \alpha) = A\cos(\frac{2\pi t}{T_0} + \alpha)$$

$$\omega = 2\pi f$$

$$T_0 = 1/f$$

Form of the sum of two complex exponential functions

$$e(t) = \frac{B}{2}e^{j(\omega t + \rho)} + \frac{B}{2}e^{-j(\omega t + \rho)} = B\cos(\omega t + \rho)$$

A, B	Amplitude
f	Frequency in hertz
α	Phase shift in radians
ω	Radian Frequency (rad/s)
T_0	Period (s)
ρ	Phase shift in radians

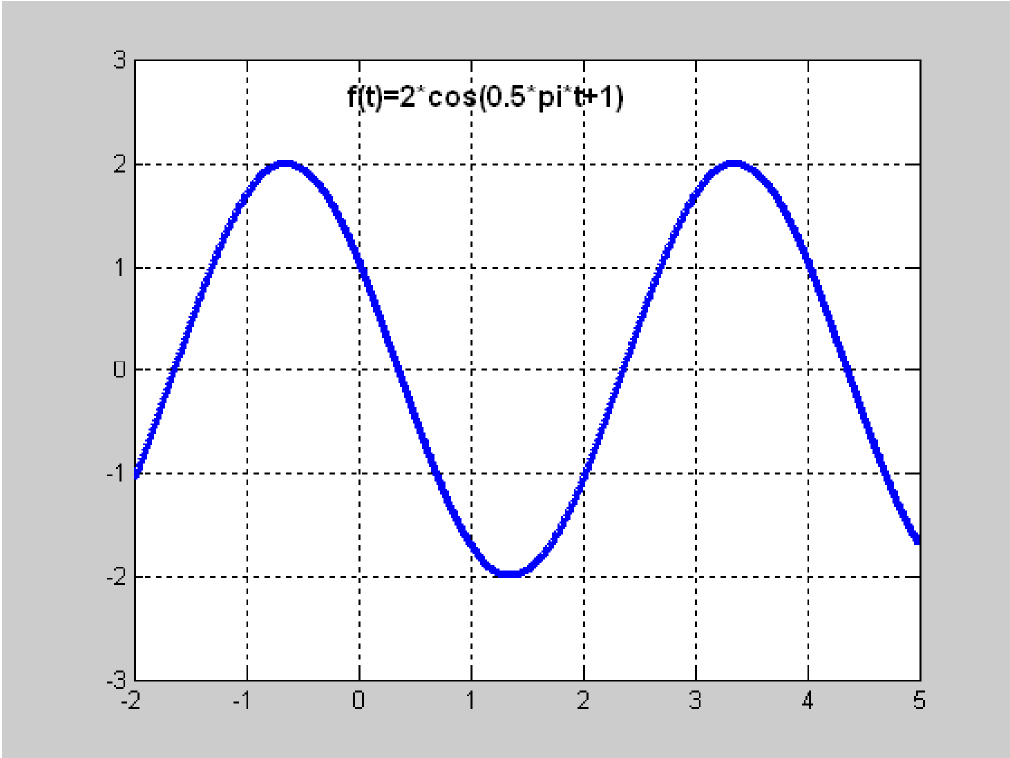
From Euler's Formula

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$g(t) = 2\cos\left(\frac{1}{2}\pi t + 1\right) = 2\cos\left(2\pi\frac{1}{4}t + 1\right) = 2\cos\left(\frac{2\pi t}{4} + 1\right)$$

$$f = \frac{1}{4} \quad \alpha = \sigma = 1 \qquad T_0 = 4$$

$$\omega = \frac{1}{2}\pi$$



Amplitude A, B

Frequency in hertz f

Phase shift in radians α

Radian Frequency (rad/s) ω

Period (s) T_0

Phase shift in radians ρ

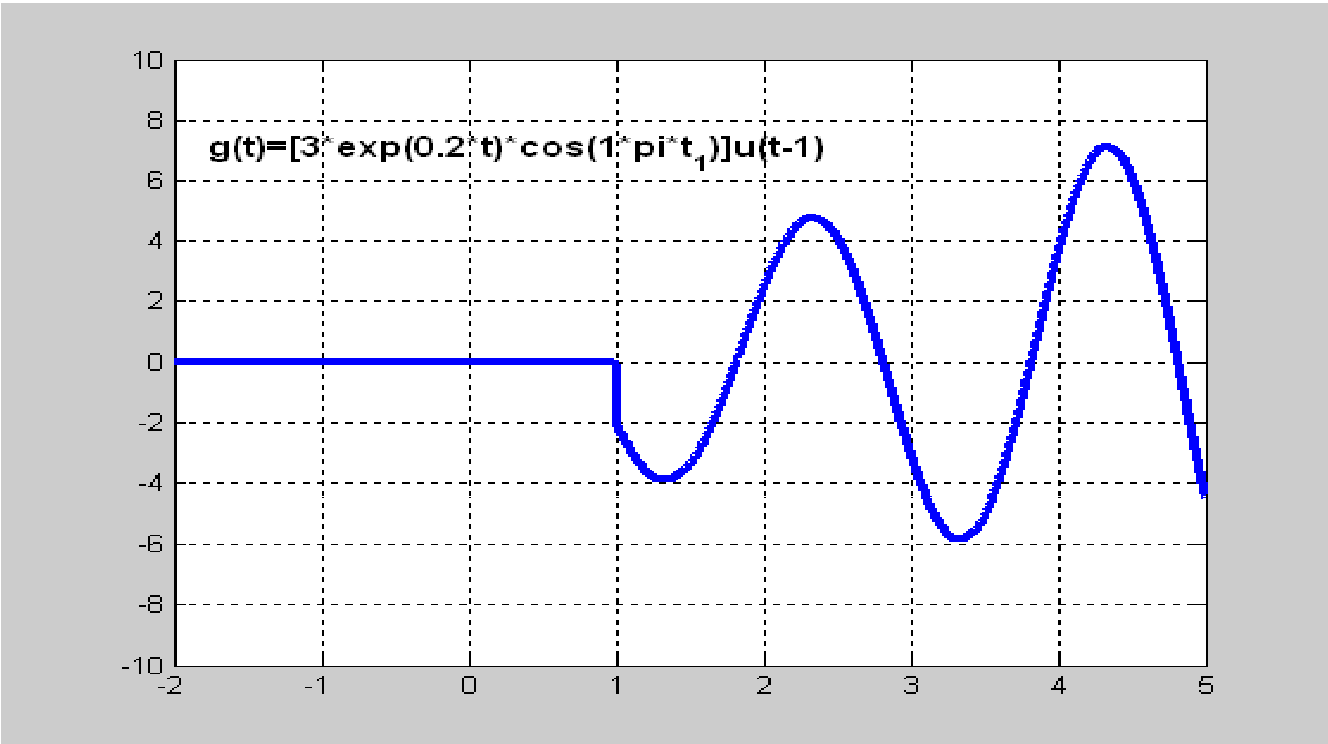
g. Exponentially modulated Sinusoidal Function

From Euler's Formula

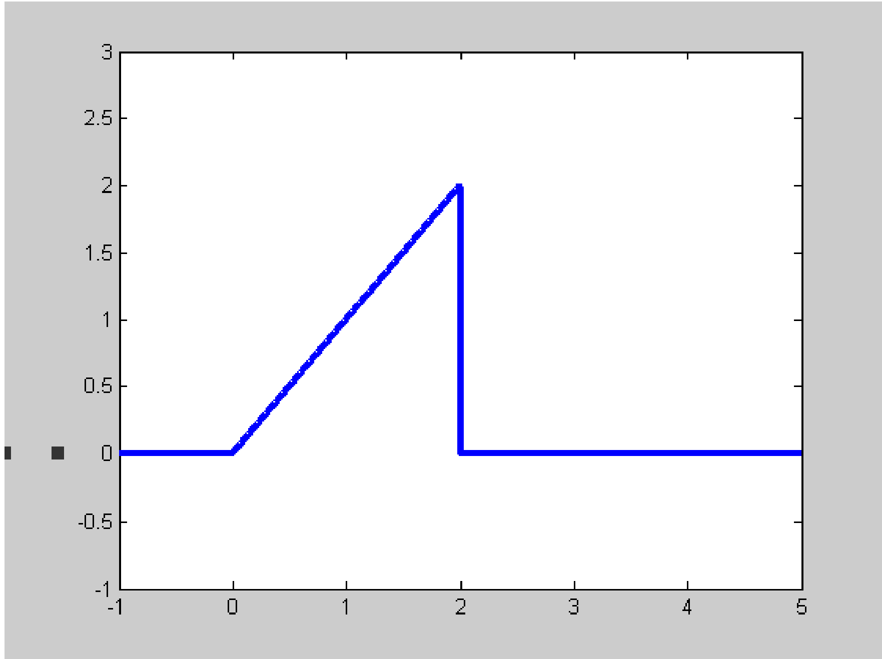
$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$f(t) = Ae^{at}\cos(2\pi ft + \alpha)$$

$$e(t) = \frac{B}{2}e^{at}e^{j(\omega t+\rho)} + \frac{B}{2}e^{at}e^{-j(\omega t+\rho)} = Be^{at}\cos(\omega t + \rho)$$

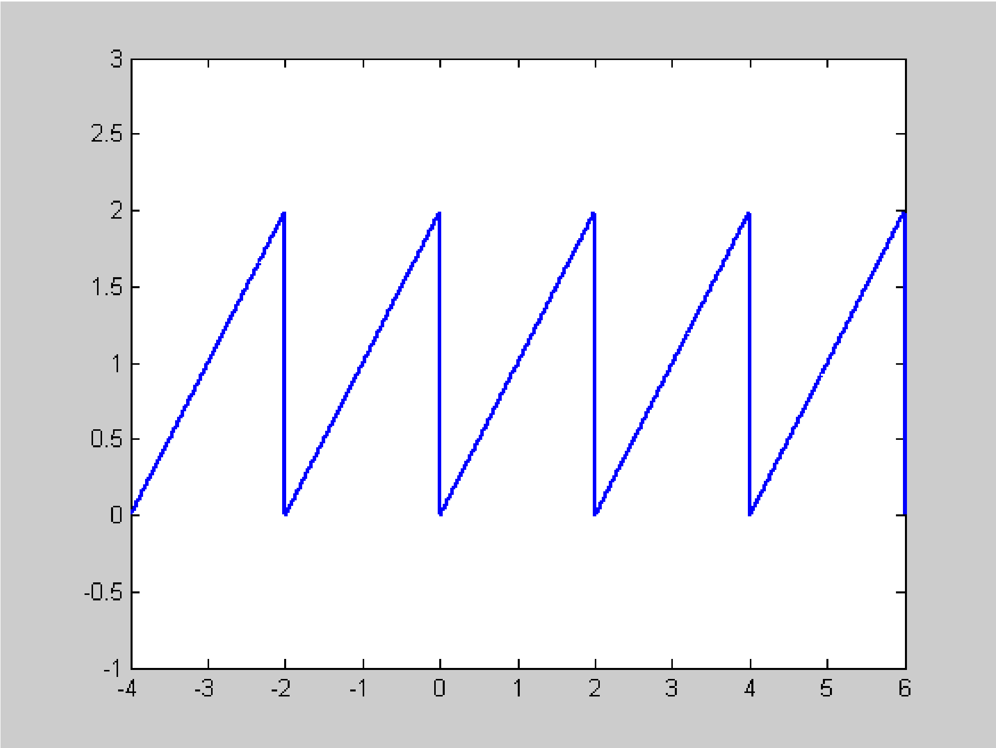


Example 1)



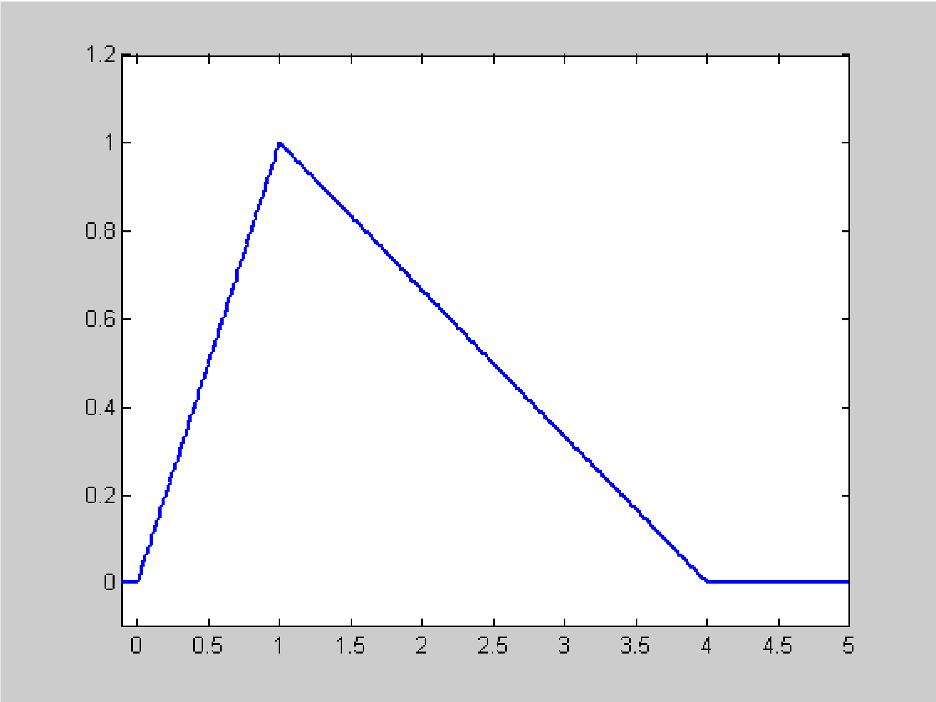
$$f(t) = t[u(t) - u(t-2)]$$

Example 2)



$$g(t)=\sum_{m=-\infty}^{\infty}(t-2m)[u(t-2m)-u(t-2m-2)]$$

Example 3)



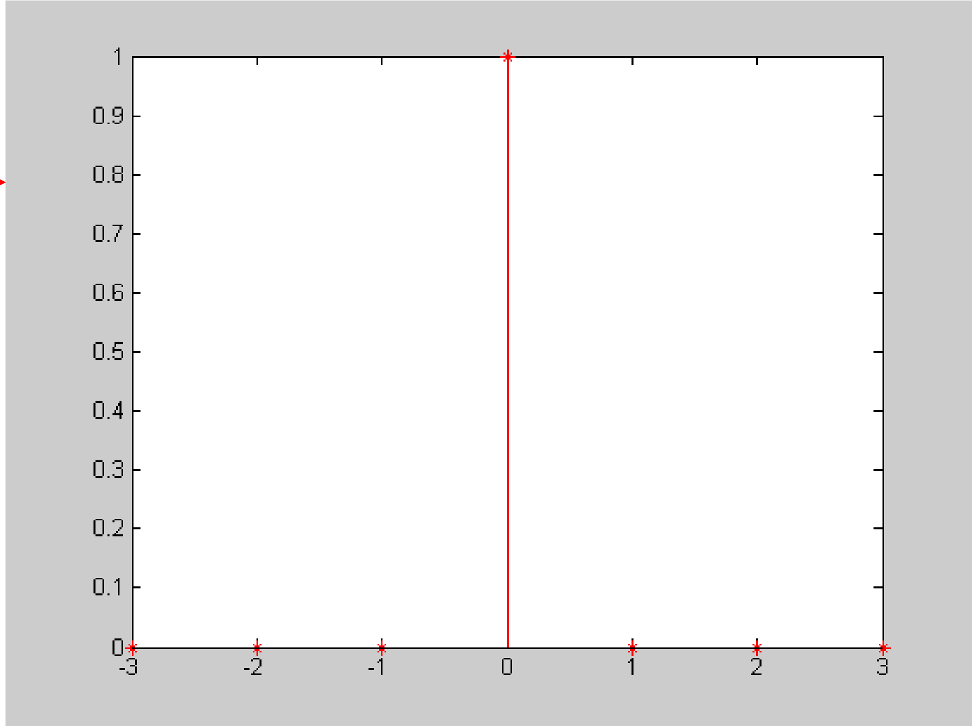
$$h(t) = t[u(t) - u(t-1)] - \frac{1}{3}[t-4][u(t-1) - u(t-4)]$$

$$= tu(t) - \frac{4}{3}[t-1]u(t-1) + \frac{1}{3}[t-4]u(t-4)$$

2. Sequences

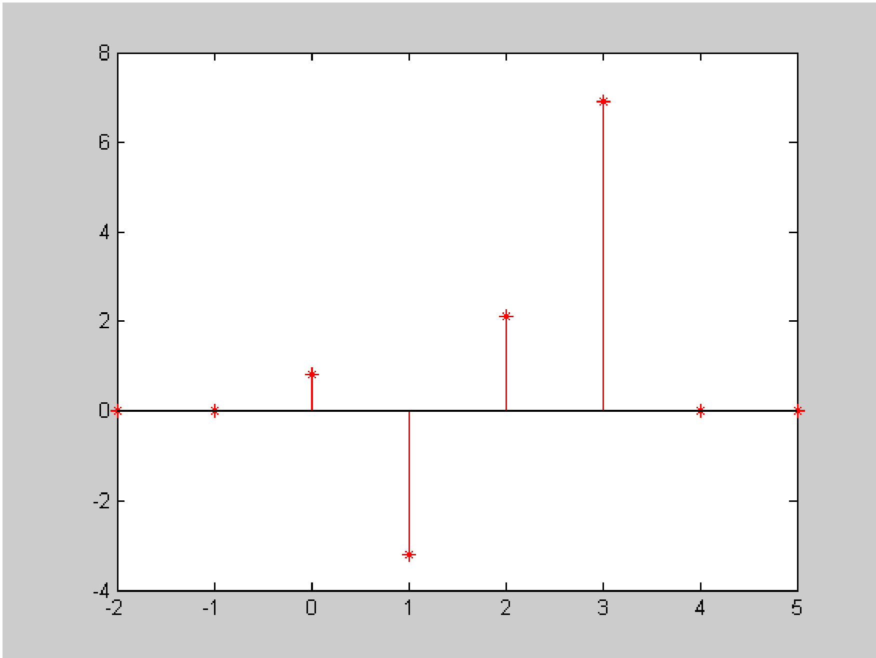
a. Unit Sample Sequences

$$\delta(m) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases}$$



b. Arbitrary Sequences

$$f(n) = \sum_{m=-\infty}^{\infty} f(m)\delta(n-m)$$

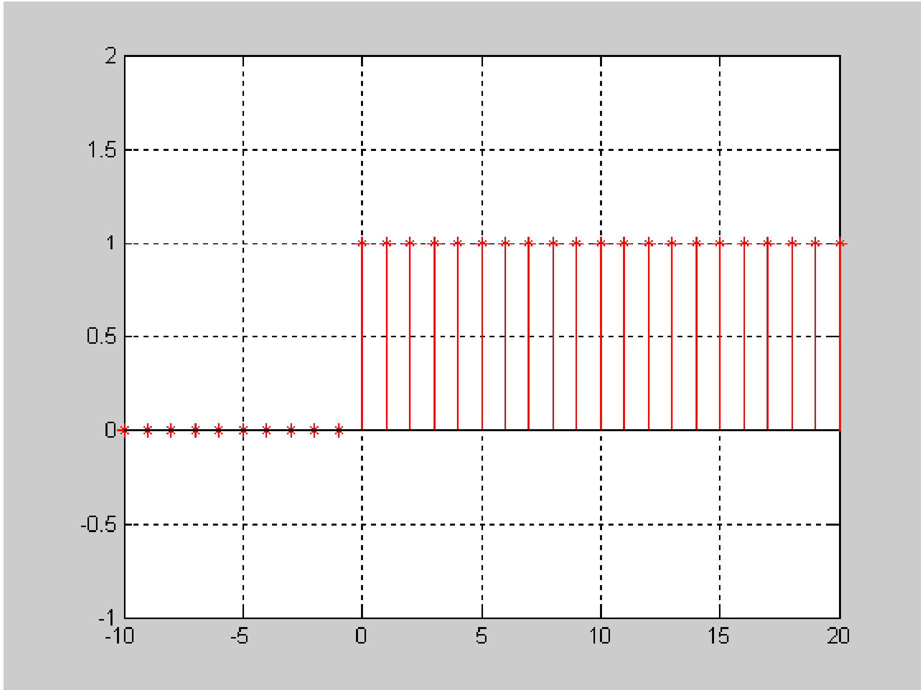


$$x(n) = 0.8\delta(n) - 3.2\delta(n-1) + 2.1\delta(n-2) + 6.9\delta(n-3)$$

c. Unit Step Sequences

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$= \sum_{m=-\infty}^{\infty} \delta(m)$$

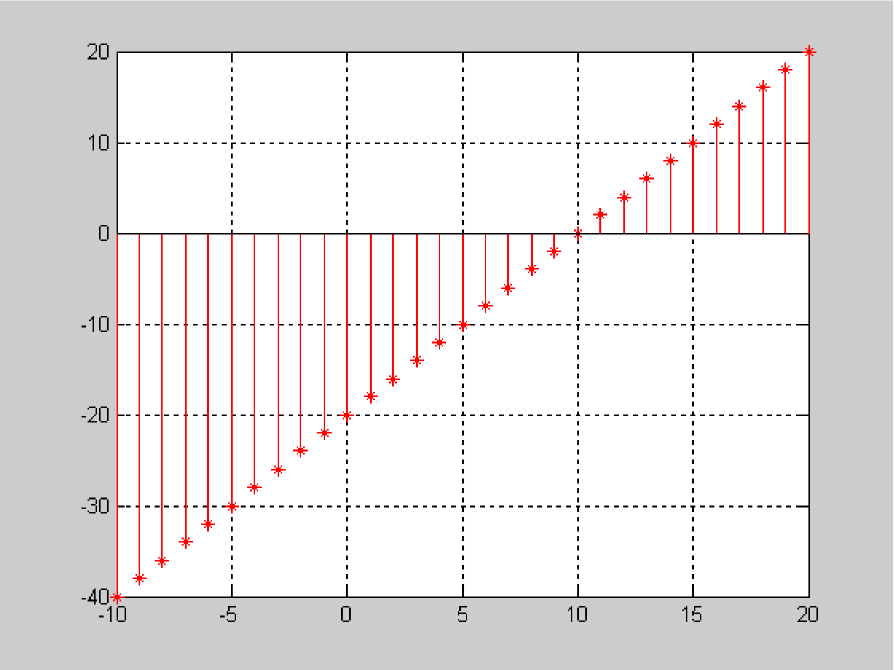


Generic

$$Bu(n - n_0) = \begin{cases} B, & n \geq n_0 \\ 0, & n < n_0 \end{cases}$$

d. Ramp Sequences

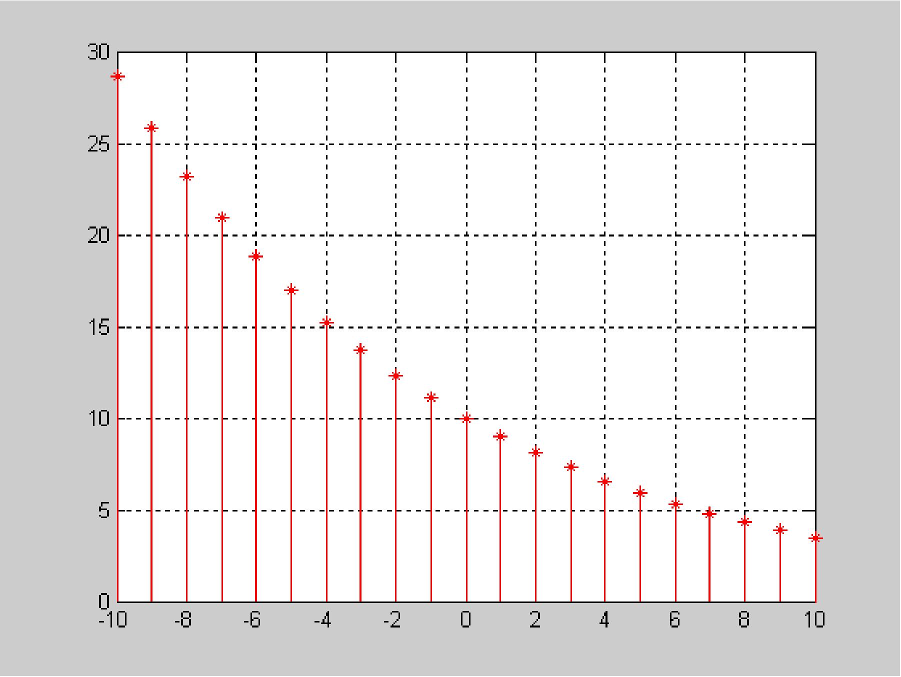
$$g(n) = B(n - n_0)$$



f(n)=2(n-10)

e. Real Exponential Sequences

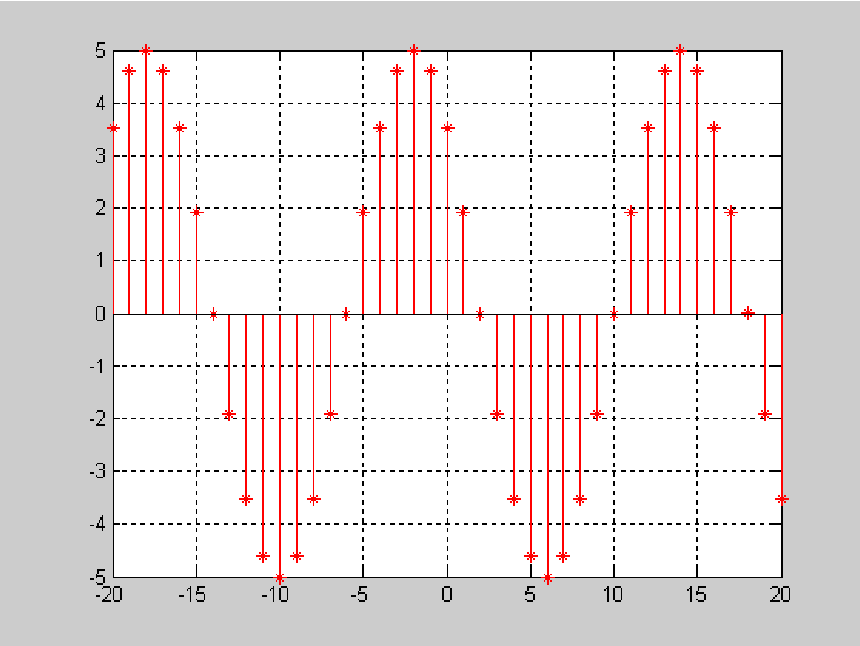
$$f(n) = A(a)^n$$



f(n)=10*(0.9)^n

f. Sinusoidal Sequences

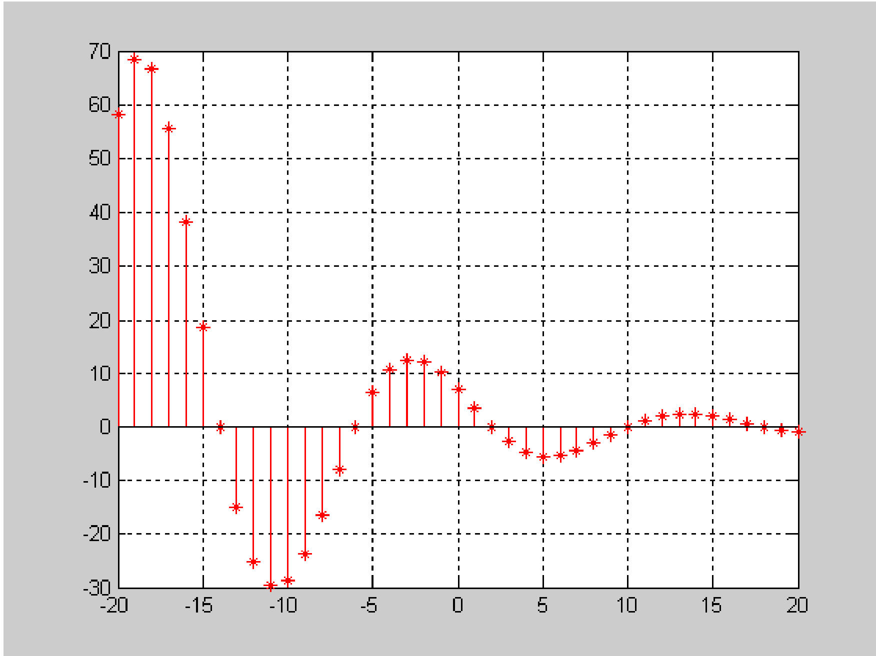
$$f(n) = A\cos\left(\frac{2\pi n}{N} + \alpha\right)$$



f(n)=5cos(2*pi*n/16+pi/4)

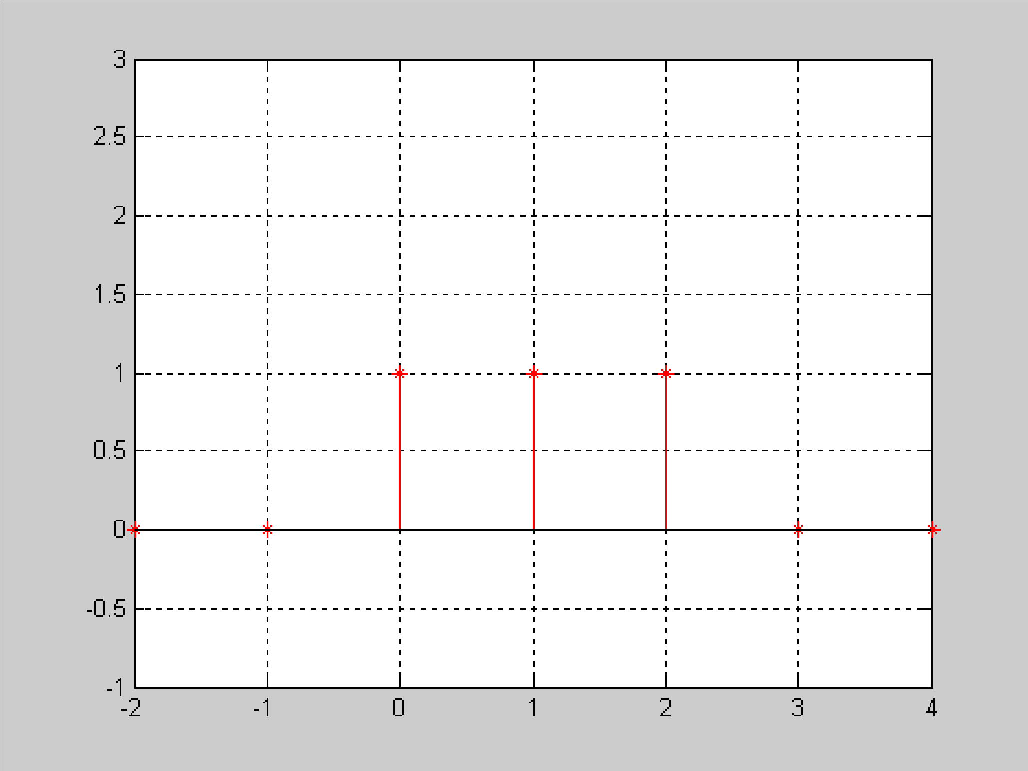
e. Exponentially Modulated Sinusoidal Sequences

$$g(n) = A(a)^n \cos\left(\frac{2\pi n}{N} + \alpha\right)$$



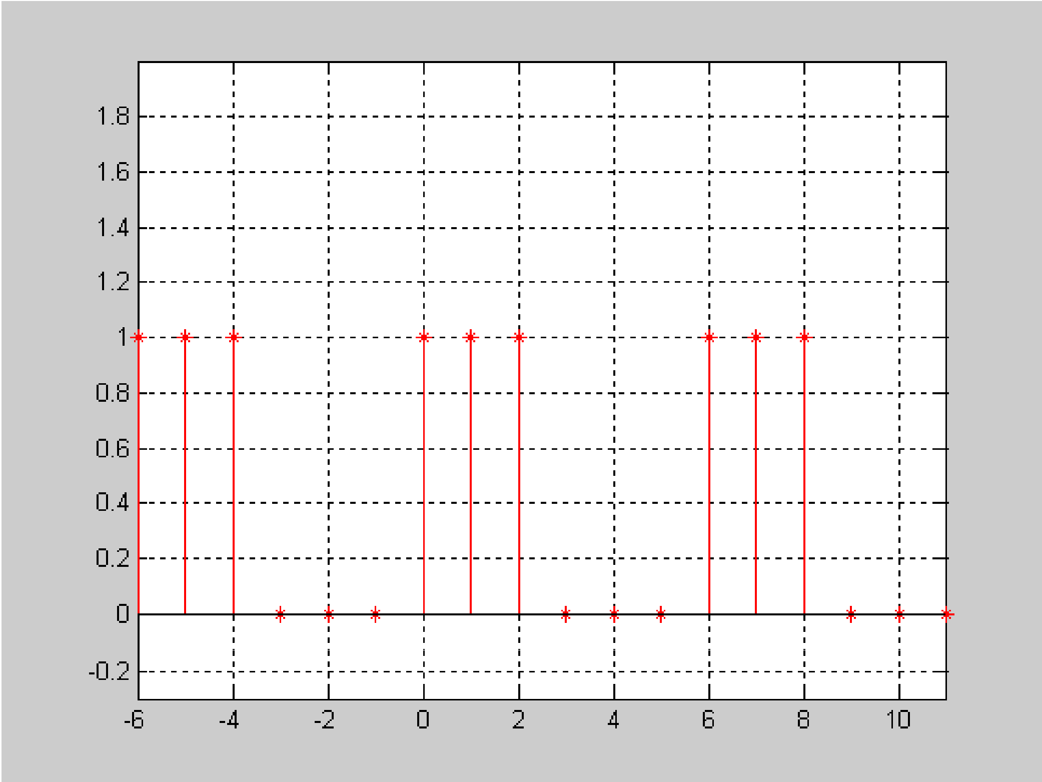
g(n)=10[(0.9)^n]cos(2*pi*n/16+pi/4)

Example 4)



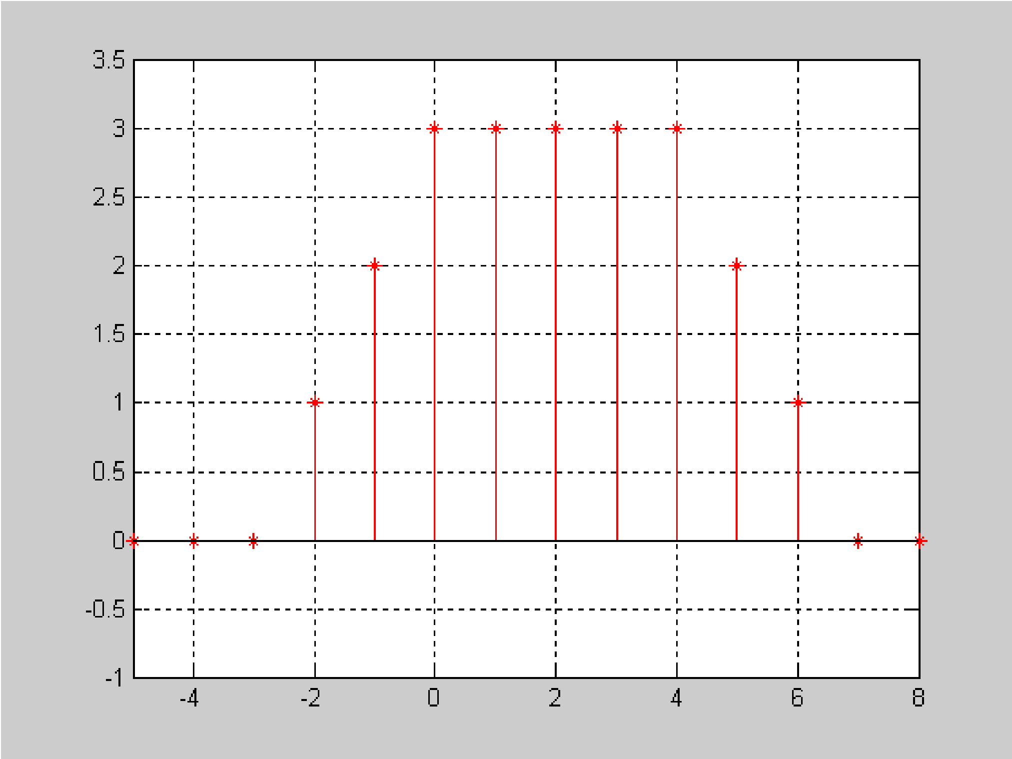
$$f_1(n) = u(n) - u(n-3)$$

Example 5)



$$f_2 = \sum_{m=-\infty}^{\infty} f_1(n-6m)$$

Example 6)



$$f_3 = (n+3)[u(n+2)-u(n)] + 3[u(n)-u(n-5)] + (-n+7)[u(n-5)-u(n-7)]$$
$$= (n+3)u(n+2) - n(u(n) - (n-4)u(n-5) + (n-7)u(n-7)$$