A Flexible New Technique for Camera Calibration

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Outline

• Camera Calibration Technique
• Basic Equations
• Solving intrinsic parameters
• Solving extrinsic parameters
• Dealing with radial distortion
• Experimental Result
• Conclusion
Camera Calibration Technique

- Photogrammetric calibration
  - Calibration Object consists of two or three planes orthogonal
- Self-Calibration
  - Moving a camera in a static scene
The Proposed Technique

• Camera to observe a planar pattern
  – At least two orientation
  – The planar pattern can move by hand
• The Technique
  – Between photogrammetric calibration
  – And self-calibration
Basic Equations

\[ s \tilde{m} = A [R \quad t] \tilde{M} \]

\[
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix} = A[R \quad t]
\begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]

\[ A = \begin{bmatrix}
  \alpha & \gamma & u_0 \\
  0 & \beta & v_0 \\
  0 & 0 & 1
\end{bmatrix} \]
Homography

• $Z=0$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A[R \quad t] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}.$$
Estimated Homography

- Maximum Likelihood Criterion
  - Levenberg-Marquardt Algorithms
  - Steepest descent method
  - Newton Method

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \| m_{i,j} - \hat{m}(A, R_i, t_i, M_j) \|^2
\]

\[
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix}
= A \begin{bmatrix}
  R \\
  t
\end{bmatrix}
\]

A, [R t] unknown
Solving intrinsic parameters

- Constraints intrinsic parameters

\[ H = A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \]

\[ \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \lambda A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \]

\[ r_1 \text{ and } r_2 \text{ are orthonormal} \]

\[ h_1^T A^{-T} A^{-1} h_2 = 0 \]
Solving intrinsic parameters

- **Singular Value Decomposition, SVD**

\[
H = A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
B = A^{-T}A^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{v_0\gamma-u_0\beta}{\alpha^2\beta^2} \\
-\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & -\frac{v_0\gamma-u_0\beta}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} \\
\frac{v_0\gamma-u_0\beta}{\alpha^2\beta^2} & -\frac{v_0\gamma-u_0\beta}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} & \frac{(v_0\gamma-u_0\beta)^2}{\alpha^2\beta^2} + \frac{v_0^2}{\beta^2} + 1
\end{bmatrix}
\]

\[
h_i^T A^{-T} A^{-1} h_2 = 0
\]

\[
h_i^T B h_j = v_{ij}^T b
\]

\[
V b = 0
\]
Solving intrinsic parameters

\[ V_b = 0 \]

\[
\begin{bmatrix}
  h_{11} h_{21} \\
  h_{11} h_{22} + h_{12} h_{21} \\
  h_{12} h_{22} \\
  h_{13} h_{21} + h_{11} h_{23} \\
  h_{13} h_{22} + h_{12} h_{23} \\
  h_{13} h_{23}
\end{bmatrix}
\begin{bmatrix}
  B_{11} \\
  B_{12} \\
  B_{13} \\
  B_{21} \\
  B_{22} \\
  B_{23} \\
  B_{33}
\end{bmatrix}
= O
\]

\[ [h_1 \ h_2 \ h_3] \]
Solving intrinsic parameters

\[
B = A^{-T}A^{-1} = \begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{12} & B_{22} & B_{23} \\
B_{13} & B_{23} & B_{33}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{\nu_0\gamma - \nu_0\beta}{\alpha^2\beta} \\
-\frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & \frac{\gamma^2}{\alpha^2\beta^2} - \frac{\nu_0\beta}{\beta^2} & \frac{\gamma(\nu_0\gamma - \nu_0\beta)}{\alpha^2\beta^2} - \frac{\nu_0}{\beta^2} \\
\frac{\nu_0\gamma - \nu_0\beta}{\alpha^2\beta} & -\frac{\gamma(\nu_0\gamma - \nu_0\beta)}{\alpha^2\beta^2} - \frac{\nu_0}{\beta^2} & \frac{(\nu_0\gamma - \nu_0\beta)^2}{\alpha^2\beta^2} + \frac{\nu_0^2}{\beta^2} + 1
\end{bmatrix}
\]

\[
\nu_0 = (B_{12}B_{13} - B_{11}B_{23})/(B_{11}B_{22} - B_{12}^2)
\]

\[
\lambda = B_{33} - [B_{13} + \nu_0(B_{12}B_{13} - B_{11}B_{23})]/B_{11}
\]

\[
\alpha = \sqrt{\lambda/B_{11}}
\]

\[
\beta = \sqrt{\lambda B_{11}/(B_{11}B_{22} - B_{12}^2)}
\]

\[
\gamma = -B_{12}\alpha^2\beta/\lambda
\]

\[
u_0 = \gamma\nu_0/\beta - B_{13}\alpha^2/\lambda
\].
Solving extrinsic parameters

\[ H = A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \]

\[ \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \lambda A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \]

\[ r_1 = \lambda A^{-1} h_1 \]
\[ r_2 = \lambda A^{-1} h_2 \]
\[ r_3 = r_1 \times r_2 \]
\[ t = \lambda A^{-1} h_3 \]
Dealing with radial distortion

- \( x, y \) : ideal normalized image coordinates
- \( x, y \) bar : real normalized image coordinates

\[
\tilde{x} = x + x[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2] \\
\tilde{y} = y + y[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]
\]

- \( u, v \) : ideal pixel image coordinates
- \( u, v \) bar : real observe image coordinates

\[
\tilde{u} = u + (u - u_0)[k_1(x^2 - y^2) + k_2(x^2 + y^2)^2] \\
\tilde{v} = v + (v - v_0)[k_1(x^2 + y^2) + k_2(x^2 + y^2)^2]
\]
Dealing with radial distortion

\[
\begin{bmatrix}
(u-u_0)(x^2+y^2) & (u-u_0)(x^2+y^2)^2 \\
(v-v_0)(x^2+y^2) & (v-v_0)(x^2+y^2)^2
\end{bmatrix}
\begin{bmatrix}
k_1 \\
k_2
\end{bmatrix} =
\begin{bmatrix}
\bar{u}-u \\
\bar{v}-v
\end{bmatrix}
\]

\[
k = (D^T D)^{-1} D^T d
\]
Complete Maximum Likelihood Estimation

$$
\sum_{i=1}^{n} \sum_{j=1}^{m} \| m_{ij} - \tilde{m}(A, k_1, k_2, R_i, t_i, M_j) \|^2
$$
Summary

- Recommended calibration procedure
  1) Print a pattern
     - attach it to a planar surface
  2) Under different orientation
     - by moving plane or camera
  3) Detect feature point
  4) Estimate intrinsic and extrinsic parameters
  5) Estimate coefficient of radial distortion
  6) Minimizing Maximum Likelihood Estimation
Camera Calibration
Experimental Result

- 8x8 squares, 256 corners
- Resolution 640x480
- Pattern size 17cm x 17cm

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<th>nb</th>
<th>initial</th>
<th>final</th>
<th>(\sigma)</th>
<th>initial</th>
<th>final</th>
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<td>825.59</td>
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<td>(k_1)</td>
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<td>0.006</td>
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<td>-0.229</td>
<td>0.006</td>
<td>0.145</td>
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<td>0.136</td>
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<td>0.881</td>
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Application to image-base modeling

Two reconstructed planes
94.7°
Conclusion

• A few different orientations
• We can move either camera or planar pattern
• Motion doesn’t need to be known
• Consists of a closed-form solution
• Based on maximum likelihood criterion